An Analysis of the Quasi Biennial Oscillation, Ozone and Thermal Damping

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I. INTRODUCTION

The stratosphere is the layer of the atmosphere starting near 10km and extending to around 50km above sea level, depending on the latitude. The factor that distinguishes the stratosphere from the troposphere which borders below and the mesosphere above, is its positive lapse rate in temperature (a positive lapse rate in temperature means that temperature increases with height which is shown in figure 1). This positive lapse rate is caused by ozone absorbing ultraviolet radiation from the sun. Unlike the troposphere, where the poles have the lowest temperatures, in the stratosphere the maximum temperature is near the summer pole and the minimum temperature is near the equator. This is a result of the ozone absorbing UV rays and the fact that there is more sunlight at the summer pole than anywhere else. This temperature pattern causes flows to be different in the stratosphere than in the troposphere. There isn't a year-round westerly (meaning winds come from the west and go east) jet in the stratosphere as there is in the troposphere. Instead, there is a westerly jet in the winter and an easterly jet in the summer. Besides the differences in the jets occurring in the troposphere and the stratosphere, there are also wind patterns unique to the stratosphere. Two important ones are the main stratospheric oscillations above the equator: the quasi-biennial oscillation (QBO), and the semi-annual oscillation (SAO). Both of these are oscillations of the zonal winds (winds that go east and west), and they occur at different altitudes in the stratosphere. The QBO is in the lower stratosphere whereas the SAO is in the upper stratosphere, and extends into the mesosphere (the mesosphere, as can be seen in figure 1, is the layer above the stratosphere).

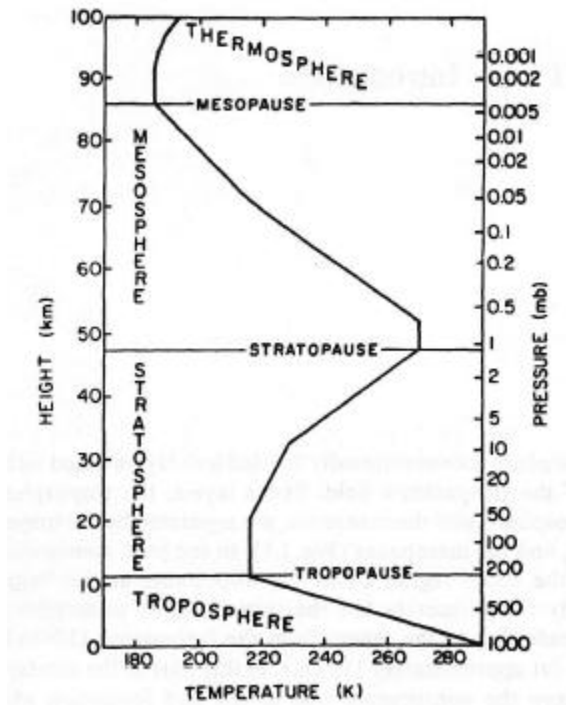

Figure 1: Temperature Profile of the Atmosphere[1]

The QBO is an oscillation of the mean zonal winds in the lower equatorial stratosphere that has a pattern shown in figure 2 (see page 4). It has maximum amplitude at the equator, around which the amplitude diminishes in a Gaussian pattern with a half-width of about fifteen degrees in latitude. The QBO stands out because it is one of the few oscillations in the atmosphere that cannot be explained by seasonal forcing patterns. Instead, the QBO is explained by the damping of vertically propagating Kelvin and Rossby-Gravity waves that give zonal momentum

[1] Andrews, Holton and Leovy 1987 p. 4

to the surrounding air parcels as they dissipate. This damping can be thermal or mechanical, and usually either occurs through radiative cooling or through disturbance by eddies or other turbulent motions in the stratosphere. In either case, a new regime (for example, a westerly flow) starts out in the stratosphere at about 35km and then propagates downward to about 23km after which the regime largely dissipates away. The damping of Kelvin waves is responsible for the westerly regimes of the QBO whereas the damping of Rossby-Gravity Waves is at least partly responsible for the easterly regimes of the QBO. The downward shift of the regimes of the QBO is caused by a Doppler-shift in the frequencies (from the reference of the ground) of the waves which occurs from the velocity of the mean zonal flow shifting the frequency of the upward-propagating waves. For example, the Kelvin waves propagate upward and eastward, so when there is westerly sheer, they will be more heavily damped in these sheer zones because they will propagate more slowly and therefore there will be more time for them to be damped. Given the transfer of energy to the mean zonal winds that occurs with their damping, this will produce westerly winds at lower levels and will prevent the waves from reaching the higher levels allowing for the set-up of the next regime of easterly winds. Once the regime of westerly winds comes close to the tropopause, it will quickly be damped and decay away. This propagation of wind regimes downward and their subsequent damping creates the oscillation shown by figure 2. Note how the pattern consistently repeats itself after about two years. This is the reason why it is called the quasi-biennial oscillation as it has a period of around 24 to 30 months with an average period of 27 months[2].

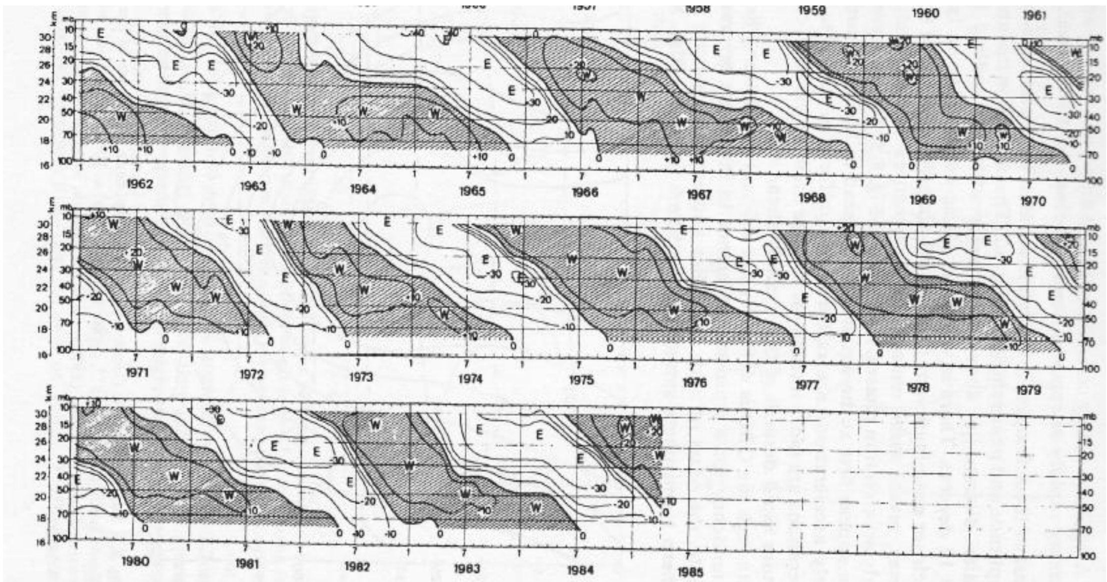

Figure 2: Observed QBO Wind Pattern[3]

While the main characteristics of the QBO have been well explored, it remains less clear how much ozone and thermal damping influence the QBO. So, in the report which follows, I will introduce more details about the background and theory behind the QBO in an attempt to set up an analysis of the effects of ozone as well as varied thermal damping have on the wind speeds, regime lengths and period of the QBO. In doing this, I intend to show how some of the characteristic features of the QBO, such as its period and wind speeds, vary as ozone is neglected and then

[2] Although the average period is 27 months (and not 24 months which would be exactly two years), there is a strong tendency for new regimes to form in the upper part of the stratosphere in the Northern Hemisphere summer which further supports the oscillation being called the QBO which would normally imply a two year period.
[3] Andrews, Holton and Leovy 1987 p.315

taken into account. This will be done in a model atmosphere where parameter values will be specified at the beginning of each simulation. After the analysis of ozone, the model atmosphere will then be modified by varying the amount of thermal damping.

II. BACKGROUND

The QBO and its characteristics have been found to be caused by a number of atmospheric wave types[4]. First, an inertia-gravity wave is a wave caused by buoyancy and of large enough wavelength such that the Coriolis force will have a significant effect. Inertia-gravity waves typically result when an air parcel is displaced vertically and horizontally. In a stably-stratified atmosphere, such a displacement causes there to be a buoyancy force on the air parcel which acts as the restoring force and causes high pressure fronts to propagate upward and horizontally as a result of the impact of the disturbed air parcel on the surrounding air parcels. Second, a Rossby wave, or planetary wave, is an oscillation of the winds resulting from meridional velocity perturbations that cause the Coriolis parameter to shift. This causes there to be a change in relative vorticity which in turn creates meridional winds north and south to account for the conservation of absolute vorticity.

Aside from these basic wave types, there are the actual equatorial waves that have been shown to cause the QBO. The first of these is the Rossby-gravity wave. A Rossby-gravity wave is a wave that resembles an internal gravity wave on

[4] Andrews, Holton and Leovy

long scales, and resembles a Rossby wave for synoptic scale motion (the synoptic scale refers to lengths on the order of of 10^6 meters). The next wave type that contributes to the QBO is the vertically-propagating Kelvin wave. A vertically-propagating Kelvin wave is a wave that propagates upward and horizontally near the equator in a similar fashion to a pure internal gravity wave and has lines of constant phase that slope upward and eastward. A key feature of the Kelvin wave is that it lacks any meridional wind perturbations. Both Rossby-gravity waves and Kelvin waves have been shown to be forced by the instability caused when there is tropical heating of cumulus clouds near the equator[5]. This instability causes compressions that propagate upward as longitudinal waves (that is, waves where particle motion occurs along the same axis as wave propagation), which propagate up into the stratosphere where they begin to get damped. This damping is the origin of the QBO and is the beginning of the process described earlier in the introduction.

In analyzing the QBO, one needs to consider the effect of ozone. First, there is a reason for the consideration of ozone and its effect on the QBO. The original model that I outlined above based solely on the damping of Kelvin and Rossby-Gravity waves has been shown to only be accurate if the amplitudes of such waves were set to be unreasonably high[6]. This means that there must be something else that contributes. Ozone would be a candidate, given that ninety percent of the

[5] Andrews, Holton and Leovy
[6] Cordero, Nathan, and Echols 1998

ozone in the atmosphere is found in the stratosphere[7] with a maximum concentration at around 22km altitude[8]. Moreover, the ozone in the stratosphere is concentrated in the equatorial region which is the same region as the QBO. This shows why ozone should be taken into consideration if one wants to get an accurate quantitative model. More importantly, ozone has the unique property of absorbing ultraviolet (UV) radiation and converting the energy of the UV radiation into thermal energy. This characteristic causes it to affect thermal damping where it impacts the QBO.

The main area of interest about the effect of ozone on the QBO is whether it could affect the ways the upward propagating Kelvin and Rossby-Gravity waves are damped. In their 1998 paper, Cordero, Nathan and Echols raise the idea that ozone heating could have some effect on thermal damping. As stated earlier, the thermal damping of Kelvin and Rossby-Gravity waves is a major cause of the QBO. In addition, Cordero, Echols and Nathan also mention that others have found that ozone has a significant impact on many other wave properties such as the stability of Rossby waves. All of this brings up the question of the importance of ozone in the QBO. In fact, the importance of ozone in the QBO is dependent on altitude. In their report, Cordero, Nathan and Echols point out that ozone begins to have a significant effect on the QBO above 35km.

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[7] Todaro
[8] Andrews, Holton and Leovy 1987

This mention of the effect of ozone on the thermal damping and its possible importance in the QBO merits an explanation of what exactly thermal damping is and how exactly ozone could affect this process. To understand the effect that thermal damping has on a wave, we need to recall LeChatlier's Principle: When a chemical system at equilibrium is disturbed, it will undergo a net change to reduce that disturbance and return to equilibrium. Although, when dealing with waves, we are not talking about a chemical system, the same principle holds. In the case of an upward propagating Kelvin or Rossby-Gravity wave, thermal damping begins when the oscillating gas molecules are heated in a diabatic process. In order to reduce the effect of the disturbance, the upward propagating wave must give up some of its energy. This is done both through the radiation of heat from the wave to the environment as well as through the transfer of some of the wave's momentum to the zonal mean flow. This transfer of momentum to the zonal mean flow gives rise to the zonal winds that comprise the QBO, and is the primary explanation of the QBO at the current time.

Related to the idea of thermal damping is the idea of Eliassen-Palm flux, or more commonly referred to as EP flux. Before discussing EP flux, it is critical to understand what an eddy is in the context of the atmosphere. In the atmosphere, an eddy is a longitudinally varying disturbance[9]. Kelvin waves and Rossby-Gravity waves

[9] Holton 2004

are eddies. EP flux gives the zonal force per unit mass of waves[10], and so it can tell us how much force an upward propagating wave exerts on the surrounding atmosphere in the zonal direction. Moreover, the change in the EP flux with height can be seen to be related to the momentum of the zonal winds that result. So, if a wave is already damped, it has a lower amplitude. Given that a wave's energy is proportional to its amplitude squared this will mean that the wave will have less energy that it is able to transfer to the surroundings which means it will not be able to produce zonal winds of very high momentum.

The EP flux is the last main concept needed before a mathematical model can be introduced and is important because it sums up the way that waves can force zonal winds. The idea of Kelvin and Rossby-Gravity waves propagating upward and forcing out zonal winds was stated to be a central idea behind the QBO. This will come up later, but first the governing equations of the atmosphere must be introduced.

III. THEORY

In order to analyze the QBO, we must be able to create a mathematical model that takes into account all the waves that cause it. In analyzing waves it is often beneficial to use perturbation theory to make a model. There are two main reasons for this: first, even after scale analysis[11] has been performed, many of the governing equations are difficult, if not impossible, to solve analytically; second,

[10] Holton 2004

[11] An example is included later in the theory section.

perturbation theory is useful because it has a form that can approximate the behavior of a wave by considering it to be in a stable base state with a small perturbative portion that may vary in any direction and with time. In general, a wave in the atmosphere is the propagation of higher pressure fronts (which would be represented by the perturbative portion in the approximation) that propagate away from the initial disturbance. A wave in the atmosphere does not cause any flow, and the pressure fronts that are created usually are not very much different than the surrounding pressure (which is what the base state would represent). This allows for the use of perturbation theory for our analysis.

In linear perturbation theory analysis, we model a variable as being the sum of a base state which is independent of time and latitude and a perturbation. The perturbation is a local variance of the field from the base state, of which is small enough that the product of any two perturbation terms is negligible relative to the base state. An example of this would be $\rho(x, y, z, t) = \bar{\rho} + \rho'$ where $\bar{\rho}(y, z)$ is the base state and $\rho'(x, y, z, t)$ is the perturbation. We note that this perturbation is the disturbance that the wave creates as it propagates through the parcel of air we are analyzing. Before we use perturbation theory on a system of equations, we are usually faced with nonlinear partial differential equations (PDEs) which are often unsolvable. Most often perturbation theory will allow these to be linearized and sometimes made into ordinary differential equations (ODEs). These are almost always solvable. From here, the goal becomes to make substitutions and eliminate variables such that we can attain one ordinary differential equation, or a single PDE of a known solvable form, and then find the time variance, zonal variance, meridional variance or vertical variance for one of the variables[12]. The purpose of this method is to determine the basic behavior of the property that is being investigated not to get an exact prediction. So, it is a trade-off. We gain a way of finding patterns and can more easily see the effect of certain parameters on the final result, but we lose the numerical accuracy that only the full partial differential governing equations can give us if solved explicitly.

An example of the use of linear perturbation theory in analyzing waves is in the analysis of a one-dimensional sound wave[13]. In doing this example, only work relating to using perturbation theory to simplify the equations is shown in an effort to demonstrate how perturbation theory can be useful in linearizing systems and finding approximate solutions. In the sound wave example, we will begin with the zonal momentum equation (1.1a) and a simplified version of the thermodynamic equation (1.1b)[14]:

$$\frac{Du}{Dt} + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0 \text{ (1.1a)}, \qquad \frac{1}{\gamma}\frac{Dp}{Dt} + p \cdot \frac{\partial u}{\partial x} = 0 \text{ (1.1b)}$$

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[12] Moreover, almost always when we use perturbation theory successfully on a system we will gain linear equations which is why the resultant equations once perturbation theory has been performed are "linearized." This is also mathematically significant because linear equations are far more easily solved.

[13] see Holton 2004 p. 189-192

[14] Note: D/Dt terms refer to the total derivative with respect to time $\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{U} \cdot \vec{\nabla}$ where $\vec{U}$ is the mean flow velocity. These have two parts: the local change (partial derivative with respect to time), and the advection part (velocity vector dot product with the gradient of whatever quantity is being differentiated).

In these equations, the variables are u=zonal velocity, ρ=density, p=pressure, γ =ratio of specific heats for air, x =zonal distance, t=time. From here we will rewrite our variables with linear perturbation theory. This is allowable for sound waves because the pressure perturbations caused by the sound waves are of insignificant in magnitude when compared to the ambient (or basic state) of the pressure. Given that the velocity and density perturbations are directly dependent on the pressure perturbations, it follows that these perturbation are also of insignificant magnitude compared to their basic states (which would be the mean zonal flow and the mean density at a given height). This then allows for the approximation (where "primed" terms are the perturbations and "barred" terms are the basic states):

$$u(x,t) = \bar{u} + u'(x,t), \quad p(x,t) = \bar{p} + p'(x,t), \quad \rho(x,t) = \bar{\rho} + \rho'(x,t)$$

We will then insert them into the governing equation (1.1a)

$$\frac{\partial}{\partial t}(\bar{u} + u') + (\bar{u} + u')\frac{\partial}{\partial x}(\bar{u} + u') + \frac{1}{(\bar{\rho} + \rho')}\frac{\partial}{\partial x}(\bar{p} + p') = 0$$

From here we use a binomial expansion so that we may eliminate $\rho'$from the third term by the fact that $\frac{\rho'}{\bar{\rho}} \ll 1$. This last step is a consequence of perturbation theory. The next step is to recognize basic states are constant with respect to zonal position and time. This causes $\frac{\partial \bar{u}}{\partial t} = \frac{\partial \bar{u}}{\partial x} = \frac{\partial \bar{p}}{\partial x} = 0$, and further simplifies the expression. Lastly, we use the fact that any product of perturbation terms is negligible to eliminate $u' \frac{\partial u'}{\partial x}$. This now gives us a fully simplified equation:

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)u'+\frac{1}{\bar{\rho}}\frac{\partial p\prime}{\partial x}=0 \qquad (1.2a)$$

This is a PDE involving only two variables as opposed to its original form which involved three, and this PDE is linear as opposed to the original PDE which was non-linear. The other governing equation (1.1b) can also be simplified using perturbation method similar to this. The process involved in doing this is a simple example that demonstrates a process critical to the mathematical model behind this report. This example demonstrates that the perturbation method can be used to simplify non-linear PDEs into linear PDEs which are more easily solvable. Plus, as often happens, the solution of this equation is a complex exponential, a form that allows for the independent analysis of the amplitude and phase portions of a perturbation. This can be exploited in finding the interdependence of field variables in an oscillation, because often we can eliminate the phase portion and be left with only amplitudes that are solvable.

An example of the inter-dependence of field variables and how to find their dependence comes from the sound wave again. For this example, I intend to show how to find the amplitude of the zonal velocity perturbation in terms of the amplitude of the pressure perturbation. We begin with equation (1.2a), and the linearized form of (1.1b) which is:

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)p'+\gamma\bar{p}\frac{\partial u\prime}{\partial x}=0 \ (1.2b)$$

We then use the solution for the pressure perturbation of the above example which is: $p' = |p'| \cdot e^{ik(x-ct)}$ where $|p'|$ denotes the amplitude of the pressure perturbation. Next, we note that the velocity perturbations must be in phase with the pressure perturbations in order to form areas of more compacted air that are the high pressure fronts. This gives $u' = |u'|e^{ik(x-ct)}$ (similar to the pressure perturbation, $|u'|$ is the amplitude of the zonal velocity perturbation). From here, our goal is to find the amplitude of the zonal velocity perturbations, $|u'|$, as a function of the amplitude of the pressure perturbations, $|p'|$, and other constants. This is achieved by plugging in our functions for u' and p' into (1.2a) and (1.2b). In doing this we get:

$$-ikc|u'|e^{ik(x-ct)} + \bar{u} \cdot ike^{ik(x-ct)} + \frac{1}{\bar{\rho}}\left(ik|p'|e^{ik(x-ct)}\right) = 0 \quad (1.3a)$$

$$-ikc|p'|e^{ik(x-ct)} + \bar{u} \cdot ik|p'|e^{ik(x-ct)} + \gamma\bar{p}\left(ik|u'|e^{ik(x-ct)}\right) = 0 \quad (1.3b)$$

We could use either one of these equations to find a relationship between the amplitude of the zonal velocity perturbation and the amplitude of the pressure perturbation. However, (1.3b) has basic state variables of pressure and zonal velocity. Given these are the two quantities we are interested in, we choose to find a relation for$|u'|$ using (1.3b), and so we eliminate terms to simplify (1.3b) to:

$$|p'|(-c + \bar{u}) + \gamma\bar{p}|u'| = 0 \quad (1.4b)$$

This has solution $|u'| = |p'|\left(\frac{c-\bar{u}}{\gamma\bar{p}}\right)$. Given the fact that both u' and p' are an amplitude multiplied by the same exponential, we may generalize: $u' = \left(\frac{c-\bar{u}}{\gamma\bar{p}}\right)p'$.

This gives us a definite relation between two perturbations involving only basic states and constants.

In using perturbation theory for the middle atmosphere we will rely mainly on five of the six main governing equations. These equations are the zonal, meridional and vertical momentum equations (after scale analysis to any reasonable accuracy, the vertical momentum equation converts into the hydrostatic equation), the mass continuity equation and the thermodynamic equation. In order to simplify these equations, we must perform scale analysis. Scale analysis works by taking the general equation, and inserting values of the variables that are in the range of what we would expect based on the data we will be working with. We then evaluate the magnitude of the terms in a summation and determine which terms are insignificant based on the amount of accuracy we want. The general form of these equations after scaled to about ninety-nine percent accuracy is[15]:

$$\frac{Du}{Dt} = fv - \frac{1}{\rho}\frac{\partial p}{\partial x} \quad \text{(zonal momentum equation)}$$

$$\frac{Dv}{Dt} = -fu - \frac{1}{\rho}\frac{\partial p}{\partial y} \quad \text{(meridional momentum equation)}$$

$$\frac{\partial p}{\partial z} + \rho g = 0 \text{ (hydrostatic equation)}$$

$\vec{\nabla} \cdot \left(\rho \vec{U}\right) = 0$, where $\vec{U} = u\hat{\imath} + v\hat{\jmath} + w\hat{k}$ is the total velocity (mass continuity equation)

[15] see Holton 2004 p. 40, 42, 46, 49

$c_v \frac{DT}{Dt} + p \frac{D\alpha}{Dt} = J$, where α=specific volume, J=diabatic heating rate (thermodynamic equation)

Depending on the situation we may select some of these equations or use all of them based on whether we are concerned with propagation in all three dimensions (zonal, meridional and vertical propagation), or whether it is accurate to neglect perturbation in a particular dimension. In addition to these base equations, we can also take advantage of other properties that may or may not apply to the situation. In some cases the ideal gas law can be used to modify these equations with perturbation terms. We will then use the method described in the sound wave example to simplify our equations as much as possible.

One other concept that plays an important role in our use of perturbation theory to analyze the QBO is the beta plane approximation. This approximation takes advantage of the form of the Coriolis parameter, $f = 2\Omega \sin \varphi$, where Ω is the angular speed of the earth and φ is the latitude. Given that the QBO exists primarily in the equatorial region with a half-width of 15 degrees, the sine function is approximately linear. We may then define $\beta = \frac{df}{dy}$, such that $f = \beta y$. This aids to a large degree in linearizing the zonal and meridional momentum equations.

For most cases when we use perturbation theory we will have field variables with periodic nature. Thus, most of the time we will end up with terms such as

$p'(x, y, z, t) = |p'|e^{i(kx+ly+mz+\nu t)}$. It is critical to note that these waves are often directly dependent on one another. This means that each wave will have the same complex exponential term. Sometimes we may know the dependence of a perturbation term on all but one or two variables such as y and z, and be trying to find how that perturbation term depends on that one variable.

An example of this concept where we have a general solution and are trying to find how it depends on two variables is the dependence on height of a Kelvin wave as it propagates upward. For this example we assume a general form for perturbations of[16]:

$$(u', v', w', \Phi') = (\hat{u}(y, z), \hat{v}(y, z), \hat{w}(y, z), \hat{\Phi}(y, z))e^{i(kx-\nu t)} \quad (2.0)$$

Next, we consider the linearized zonal momentum, meridional momentum, continuity and thermodynamic equations[17]:

$$\frac{\partial u'}{\partial t} + \bar{u}\frac{\partial u'}{\partial x} - \beta y v' = -\frac{\partial \Phi'}{\partial x} \quad (2.1)$$

$$\frac{\partial v'}{\partial t} + \bar{u}\frac{\partial v'}{\partial x} + \beta y u' = -\frac{\partial \Phi'}{\partial y} \quad (2.2)$$

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + {\rho_0}^{-1}\frac{\partial(\rho_0 w')}{\partial z} = 0 \quad (2.3)$$

$$\frac{\partial}{\partial z}\left(\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right)\Phi' + w'N^2 = 0 \quad (2.4)$$

Now, we want to simplify the third term on the left hand side of equation 2.3 (the linearized continuity equation). To do this, note that the governing equations are

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[16] Holton 2004, p.430

[17] Holton 2004, p.430. These have been modified by removing the assumption that there is no base zonal or meridional flow and so placing an extra term in the zonal and meridional momentum equations.

applicable to the basic states. This means that we may consider the hydrostatic equation in terms of ρ_0. This equation is:

$$\frac{1}{\rho_0} \cdot \frac{\partial \bar{p}}{\partial z} = -g$$

Next use the ideal gas equation to substitute $\bar{p} = \rho_0 R \bar{T}$ to get (note: Here we define $\bar{T}$ to be the mean temperature for a layer of the atmosphere and, for the moment, we neglect perturbations to it):

$$\frac{RT}{\rho_0} \cdot \frac{\partial \rho_0}{\partial z} = -g \Leftrightarrow \frac{\partial \rho_0}{\partial z} = -\frac{\rho_0 g}{R\bar{T}}$$

Now, after using a product rule and substituting this result into (2.4), we get:

$$\frac{\partial u\prime}{\partial x} + \frac{\partial v\prime}{\partial y} - \frac{w\prime g}{RT} + \frac{\partial w\prime}{\partial z} = 0 \quad (2.5)$$

At this point we note that Kelvin waves lack a meridional velocity perturbation, so $v'=0$. Next, we substitute the basic form of our perturbations, equation 2.0, into our equations and simplify. This gives:

$$-\nu\hat{u} + k\bar{u}\hat{u} = k\widehat{\Phi} \quad (2.6)$$

$$\beta y \hat{u} = \frac{\partial \widehat{\Phi}}{\partial y} \quad (2.7)$$

$$ik\hat{u} - \frac{g}{R\bar{T}}\widehat{w} + \frac{\partial \widehat{w}}{\partial z} = 0 \quad (2.8)$$

$$\frac{\partial \widehat{\Phi}}{\partial z}(-i\nu + \bar{u}ik) + N^2\widehat{w} = 0 \quad (2.9)$$

From here, we solve (2.6) for $\widehat{\Phi}$ and substitute this into (2.9) then solve for $\widehat{w}$ to get:

$$\widehat{w} = \frac{-i\nu + i\bar{u}k}{N^2} \cdot \frac{\partial}{\partial z}\left(\frac{\nu}{k} - \bar{u}\right)\hat{u} \quad (2.10)$$

Next, we substitute (2.10) into (2.8), which results in (after multiplying both sides by N^2 and dividing by i for simplification):

$$\frac{\partial^2\hat{u}}{\partial z^2}(-\nu+\bar{u}k)\left(\frac{\nu}{k}-\bar{u}\right)+\frac{\partial\hat{u}}{\partial z}(\frac{-g}{R\bar{T}}(-\nu+\bar{u}k)\left(\frac{\nu}{k}-\bar{u}\right)+N^2k\hat{u}=0 \quad (2.11)$$

Note that this is an ODE, and we can solve it by assuming $\hat{u}$ has the form $\hat{u}=A(y)\cdot e^{qz}$ where A(y) is an arbitrary function of y and q is a complex number (constant). By taking derivatives of $\hat{u}$ and substituting these into (2.11), (2.11) becomes (after cancelling the exponentials and other terms):

$$q^2(-\nu+\bar{u}k)\left(\frac{\nu}{k}-\bar{u}\right)+q\cdot\frac{-g}{R\bar{T}}(-\nu+\bar{u}k)\left(\frac{\nu}{k}-\bar{u}\right)+N^2k=0 \quad (2.12)$$

From here we can use the quadratic formula to solve for q, and we get (note: H is the scale height, H=RT/g):

$$q=\frac{1}{2H}\pm\sqrt{\left(\frac{g}{2R\bar{T}}\right)^2-\frac{N^2k^2}{(\nu-\bar{u}k)^2}}$$

This gives: $\hat{u}=A(y)e^{\frac{z}{2H}\pm\left(\sqrt{\left(\frac{g}{2R\bar{T}}\right)^2-\frac{N^2k^2}{(\nu-\bar{u}k)^2}}\right)z}$. We now note that vertical (z) dependence is only in that exponential. This means that it will appear in every field variable as the same exponential because exponentials remain intact while taking derivatives. While other constants will arise in computing the other fields, we can simply move them to the meridional dependence equations, $\hat{u}(y),\widehat{w}(y),\widehat{\Phi}(y)$. This allows us to write our perturbation fields as:

$$(u',w',\Phi')=(\hat{u}(y),\widehat{w}(y),\widehat{\Phi}(y))e^{i(kx-\nu t)}e^{\frac{z}{2H}\pm\left(\sqrt{\left(\frac{g}{2R\bar{T}}\right)^2-\frac{N^2k^2}{(\nu-\bar{u}k)^2}}\right)z} \quad (2.13).$$

And so we have found the vertical dependence for the Kelvin wave. Next rewrite (2.13):

$$(u', w', \Phi') = (\hat{u}(y), \hat{w}(y), \hat{\Phi}(y))e^{i(kx-\nu t)}e^{\frac{z}{2H}\pm\left(i\cdot\sqrt{\frac{N^2k^2}{(\nu-\bar{u}k)^2}-\left(\frac{g}{2R\bar{T}}\right)^2}\right)z} \quad (2.14)$$

Now define $m^2 \equiv \frac{N^2k^2}{(\nu-\bar{u}k)^2}$, and rewrite $\left(\frac{g}{2R\bar{T}}\right)^2 = \frac{1}{4H^2}$. From this, (2.14) becomes:

$$(u', w', \Phi') = (\hat{u}(y), \hat{w}(y), \hat{\Phi}(y))e^{i(kx-\nu t)}e^{\frac{z}{2H}\pm\left(i\cdot\sqrt{m^2-\frac{1}{4H^2}}\right)z} \quad (2.15)$$

From here we will perform scale analysis on the radical in the exponential. Vertically propagating Kelvin waves have a wavelength of 6 to 10km[18]. Thus, we may write $\lambda \sim 10^4$m. Given $m = \frac{2\pi}{\lambda}$, $m \sim 10^{-3}$m^{-1} or $m^2 \sim 10^{-6}$m^{-2}. In the stratosphere, the scale height is generally about 6km to 7km, so it is reasonable to write $H \sim 10^4$m. This makes the second term in the radical on the order of 10^{-8} m. This means the first term is generally about one hundred times larger than the second term which implies that we may neglect the second term in the radical and still have an error of only about one percent. For the case of our simulation this error is acceptable, so we may write a scaled version of (2.15) as:

$$(u', w', \Phi') = (\hat{u}(y), \hat{w}(y), \hat{\Phi}(y))e^{i(kx-\nu t)}e^{\frac{z}{2H}\pm imz} \quad (2.16)$$

The exponential involving vertical dependence can be further simplified if we consider the equation for a harmonic wave. In 2.16, m is the wave number in the vertical direction for a harmonic wave. For harmonic waves, the sign of the wave

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[18] Holton 2004

number determines the direction of phase velocity or phase propagation (shown by the fact that phase velocity, $v=\omega/k$ for $\omega=$ angular frequency, and k=wave number). For vertically propagating Kelvin waves, our phase velocity along the z-axis is downward which corresponds to a harmonic wave of the form (for simplicity excluding horizontal dependence) $f(z,t) = e^{i(-\nu t - mz)}$. Therefore, we can use the "-" sign for the "imz" term assuming m is positive. Next, we want to reinsert this into our standard form (2.16). By convention, we write $m<0$ and flip the sign to $-mz$. In the broader context, this makes it easier to compute the group velocity (velocity of the wave packet), but is done here only to fit the standard way of writing the equation. With this, 2.16 becomes:

$$(u', w', \Phi') = (\hat{u}(y), \hat{w}(y), \hat{\Phi}(y))e^{i(kx+mz-\nu t)}e^{\frac{z}{2H}} \quad (2.17)$$

At this point, the basic techniques needed to model the QBO have been shown. However, on top of these basic techniques, there are other techniques needed to account for ozone. Preceding the introduction of these techniques, one needs to know some basics about ozone.

When we consider ozone, we should begin by looking at its creation and destruction to get a basic understanding of ozone production in the stratosphere. The basic chemical reaction scheme for this was discovered by Sydney Chapman around 1930, and so the reactions are called the Chapman reactions[19]. The Chapman reactions are[20]:

$$O_2 + h\nu \rightarrow 2O \quad (C1)$$

$$O + O_2 + M \rightarrow O_3 + M \quad (C2)$$

$$O_3 + h\nu \rightarrow O + O_2 \quad (C3)$$

$$O + O_3 \rightarrow 2O_2 \quad (C4)$$

In these reactions, M refers to another molecule (typically O_2 or N_2) that is needed to absorb and balance the energy of C2. The photon, hv, refers to the ultraviolet radiation that the oxygen molecule must absorb to start the process. In the case of the separation of an O_2 molecule into two oxygen atoms (reaction C1), a wavelength in the ultraviolet (UV) range that has a wavelength of below 240nm is required[21]. Photons of such wavelength are not in abundance in the solar radiation incident on the stratosphere. This makes this reaction somewhat slow in comparison to C2. Reaction C3 is also fast and, once again, involves UV radiation as this time the radiation splits an ozone molecule into an oxygen molecule and an oxygen atom. It is critical to note that this reaction (C3) does not contribute to the loss of ozone. The reason is that the oxygen atom has a short lifetime, compared to ozone and oxygen molecules (O_2). This means that the oxygen atom that results from C3 will typically combine with an oxygen molecule from C2 to form an ozone molecule. Of these

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[19] Todaro Ch. 5
[20] Andrews, Holton and Leovy p. 401
[21] Todoro Ch. 5

reactions, ozone depletion occurs in C4 where the products (two oxygen molecules) are stable, and can only be converted to ozone through the slow reaction C1[22].

The only problem that has been found with the Chapman reactions in accounting for the creation and destruction of ozone in the stratosphere is that the destruction in C4 occurs at a slower rate than observed in the stratosphere. The cause for this inaccuracy is the presence of free radicals that act as catalysts in a multi-step reaction whose net equation fits C3 and C4 added together. In brief, such catalysts are typically chlorine, bromine and hydrogen. Often these reach the stratosphere via emission of methane gas or chlorofluorocarbons found in pollution[23]. Nonetheless, the specifics of these pollutants are outside of the scope of this project.

The next thing that must be covered is how to account for ozone in mathematical models. To do this a technique for approximations must be introduced and we must modify our fundamental equations (2.1 to 2.5) to include the effects of ozone.

First, the approximation technique needed for this is the WKB approximation. For the WKB approximation, we will assume that an oscillation has the basic form of a wave although we will allow its frequency, wavelength and amplitude to vary with time and in one or more directions. This approximation is

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[22] Andrews, Holton and Leovy p. 401
[23] Todoro Ch. 1

valid only if ozone does not exert enough influence on disturbances such as Kelvin waves and Rossby-Gravity waves to cause their wavelength to change very much within the distance of one wavelength, or mathematically[24] if $\frac{1}{2\pi}\left|\frac{d\lambda}{dx}\right| \ll 1.$ Observations have shown that even in the presence of naturally occurring ozone, Kelvin and Rossby-Gravity waves show periodic behavior with only small variation in wavelength. Thus, the WKB approximation is valid for modeling upward propagating waves in the stratosphere with ozone. To write the wave in such a form, we will then rewrite (2.17) as

$$(u', w', \Phi') = (\hat{u}(y,z), \widehat{w}(y,z), \widehat{\Phi}(y,z))e^{i(k(z)x+m(z)z-\nu(z)t)}e^{\frac{z}{2H}} \quad (2.18)$$

The inclusion of z inside the parentheses means that each term slowly varies with z, the height. This variance of amplitude and wavelength with height will be shown to be a key result of the ozone. As was stated in the introduction, ozone amounts vary significantly with height.

The second part of accounting mathematically for the addition of ozone involves modifying the governing equations. In this modification, there is one new equation, the ozone continuity equation, a new parameter, and three new coefficients. These new parameter is γ, the ozone mixing ratio. The new coefficients are the ozone heating coefficient A, the ozone relaxation coefficient B and the

[24] Fowler 2008

ozone temperature coupling coefficient C. The governing equations with ozone are[25]:

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)u' + w'\frac{\partial \bar{u}}{\partial z} - \beta y v' = -\frac{\partial \Phi'}{\partial x} \quad (3.1)$$

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)v' + \beta y u' = \frac{\partial \Phi\prime}{\partial y} \quad (3.2)$$

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \rho^{-1}\frac{\partial(\rho w')}{\partial z} = 0 \quad (3.3)$$

$$\frac{\partial \Phi\prime}{\partial z} = \frac{RT\prime}{H} \quad (3.4)$$

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)T' + v'\frac{\partial \bar{T}}{\partial y} + \frac{H}{R}N^2 w' = -\alpha T' + \frac{H}{R}A\gamma' \quad (3.5)$$

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)\gamma' + v'\frac{\partial \bar{\gamma}}{\partial y} + w'\frac{\partial \bar{\gamma}}{\partial z} = -B\gamma' - \frac{R}{H}CT' \quad (3.6)$$

Like the governing equations that did not account for ozone, 2.1 to 2.4, these are also coupled PDEs. More important is the significance of some of the terms. Equations 3.1, 3.2, and 3.3 have the same significance as stated earlier. Equation 3.4 is just another form of the hydrostatic equation.

Now consider equation 3.5. Although it may not be easily recognizable, it is similar to equation 2.4 in that both of these represent the linearized thermodynamic equation. In the case of equation 3.5, it has been modified by switching the dependence from the geopotential perturbation, Φ', to the temperature perturbation, T'. However, the most important difference between equation 2.4 and equation 3.5 is that equation 3.5 includes the effects of ozone in its last term. The

[25] Cordero, Nathan and Echols 1998

immediate implication is that the thermodynamic equation is the first place where we see ozone effects taken into account. Also note the general form in which this equation is written. This equation is written as a total derivative of temperature on the left hand side where the $\frac{H}{R}N^2w'$ represents the vertical advection part, and then there are two terms on the right hand side. These two terms represent two of the ways that temperature can be disturbed for a system like this one. First of all, there can be diabatic heating, which refers to an increase in temperature directly caused by heat energy transfer. The effect of this is represented by the term $-\alpha T'$ (recall that T' refers to the temperature perturbation and so can be considered to be a representation of the amount of diabatic heating). Next, the absorption of UV rays by ozone can increase the temperature of the system. This is accounted for by the term $\frac{H}{R}A\gamma'$. Note how this term is positive indicating that increasing the ozone perturbation mixing ratio will cause a positive change in the temperature perturbation. One way of disturbing the temperature that is not mentioned is adiabatic heating (i.e. adiabatic compression), which is represented in the vertical part of the total derivative of temperature, $\frac{H}{R}N^2w'$.

Next, consider equation 3.6. Equation 3.6 is referred to as the linearized ozone continuity equation and takes a similar form to the mass continuity equation. It is written as the total derivative of the ozone mixing ratio (i.e. the total change in the ozone mixing ratio following a parcel of air) being equivalent to the factors that can affect the ozone mixing ratio. As stated by Todoro in Chapter 3, section 2 of his online text, the ozone mixing ratio is the fractional number of air molecules that are ozone molecules for any region. This is valuable as opposed to considering the amount of ozone via concentration because the pressure difference will lead to confusingly lower concentrations of ozone at higher altitudes in the stratosphere even when ozone accounts for a higher portion of chemicals found in the air at these altitudes. This allows ozone mixing ratio to be a valuable comparison of amounts of ozone different altitudes.

In the linearized ozone continuity equation 3.6, we see that the total derivative of the ozone mixing ratio is equal to two terms: $-B\gamma'$ and $-\frac{R}{H}CT'$. Understanding the meaning of these is critical to understanding the ways that ozone will fluctuate when perturbed. That said, the term $-B\gamma'$ represents how the system will react when the amount of ozone present is changed. This can be thought of as being similar to LeChatlier's Principle whereby a system disturbed from equilibrium will shift in a way such that the change is minimized and it returns to equilibrium. With that in mind, it should make sense that this term has a minus sign because an increase in the amount of ozone will tend to cause the system to try to minimize the excess, get rid of ozone to the surroundings and return to equilibrium. The next term, $-\frac{R}{H}CT'$, shows a different way that equilibrium can be disturbed that will affect ozone concentration: a temperature change. The reason that this term exists is that the Chapman reactions (equations C1 to C4), have rates are temperature dependent. Therefore, the temperature effects how quickly ozone is created and destroyed. Here, the reactions destroying ozone have quicker rates than the reactions creating it at higher temperatures. This effect implies that a positive perturbation in temperature will tend to lower the ozone mixing ratio, which is also indicated by the fact that this term is negative.

Relating to the equations modified for ozone and the ozone continuity equation is photochemically accelerated cooling. This effect is seen to occur above 35km where ozone plays a larger role. The reason that there is photochemical control above 35km, but dynamical control below lies in the values of the ozone coefficients. Near 35km, the values of all three ozone coefficients drastically increase in value as can be seen by figure 3. These coefficients include A, the heating coefficient, B, the ozone relaxation coefficient and C, the thermal relaxation coefficient. The higher values of B and C above 35km lead to the Chapman reactions, which produce and destroy ozone, having a greater affect on the total ozone than vertical wind perturbations. The higher value of the ozone heating coefficient, A, then leads to perturbations in the concentration of ozone having more control over the temperature perturbations. All of this leads to the full scenario of photochemically accelerated cooling. First, there is a high temperature anomaly in the upper-stratosphere. This causes there a negative perturbation in the concentration of ozone given the predominance of the ozone relaxation coefficient, B, over wind perturbations. Then, less UV radiation is absorbed because there are fewer ozone molecules. Given the high amount of heat these molecules give off at this altitude by absorbing UV rays, the loss creates significantly less heat and therefore cools the area significantly.

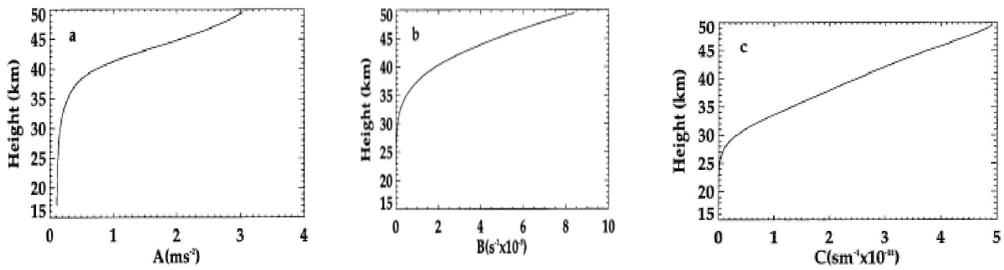


Figure 3: Distributions of the Ozone Coefficients[26]

Now that the ozone coefficients have been explained, this can be combined with what was said earlier to show a critical effect of adding ozone on the QBO. When I introduced the WKB, I made a note of the height dependence of the QBO relating to the varying amounts of ozone at different altitudes. That point is relevant here. As stated by Echols and Nathan in their 1996 paper, the components of the vertical wave number of a vertically propagating Kelvin wave modified by ozone are:

$$Re(m) = \frac{Nk}{\omega} \quad (4.1)$$

$$Im(m) = \frac{Nk}{2(\omega^2+B^2)}\left(\alpha - \frac{A\overline{\gamma_z}}{N^2} + B\left(\frac{\alpha B + AC}{\omega^2}\right)\right) \quad (4.2)$$

Given the form of (2.17), which gives the basic form of perturbations relating to the QBO, of which this applies, the impact is significant. First, we note the basic form of

[26] Cordero, Nathan and Echols 1998

a sinusoidal wave and the fact that m is the vertical wave number here with an angular frequency of ω. This makes (4.1) the dispersion relation for the resulting perturbation. Equation 4.2, however, is far more significant. Given the fact that equation 4.2 is the imaginary part of m, this term will no longer be part of an exponential to an imaginary power and therefore will no longer represent the sinusoidal part of a wave. Instead, equation 4.2 relates to the amplitude dependence of the waves. It is arguably the most important part of what I have presented for the effect of ozone because it incorporates ozone into the amplitude of these perturbed quantities. The first way is in the effect of the $\frac{A\overline{\gamma_z}}{N^2}$ term which incorporates the effect of the ozone heating into the damping of the wave. A plot of the dependence of $\overline{\gamma_z}$ with height is shown in figure 4 for clarification.

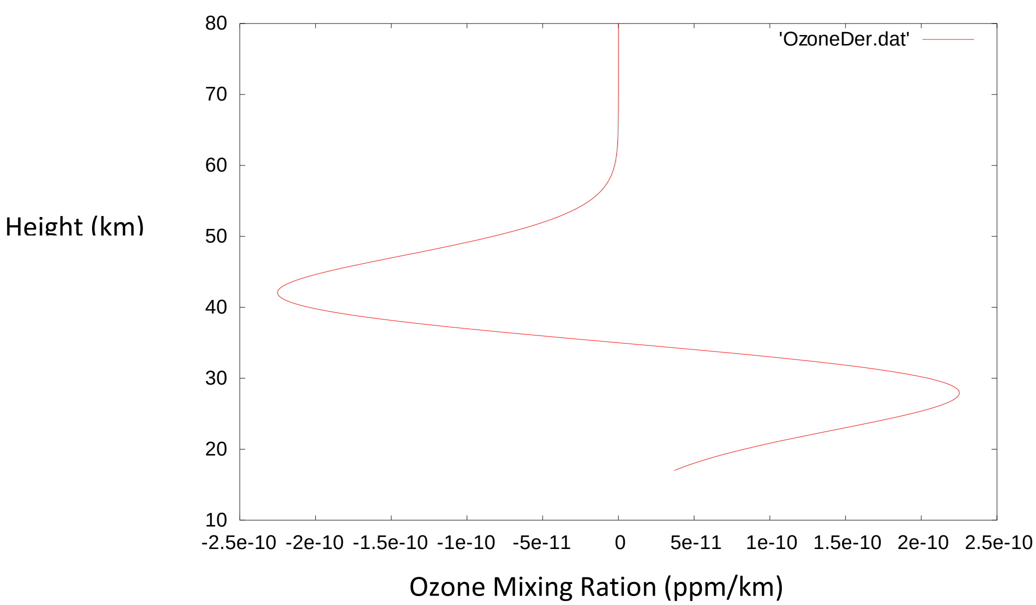
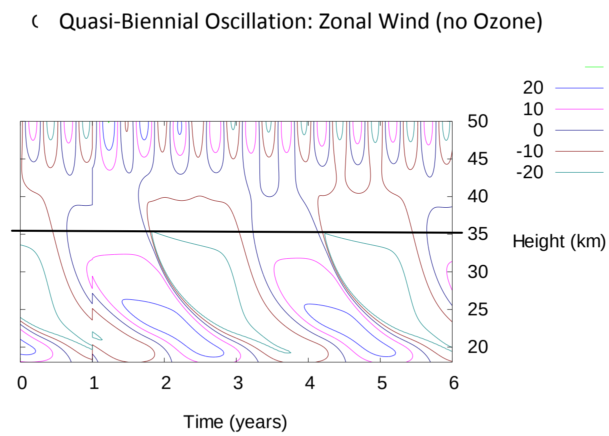


Figure 4: Height Versus the Vertical Derivative of the Ozone Mixing Ratio, $\overline{\gamma_z}$

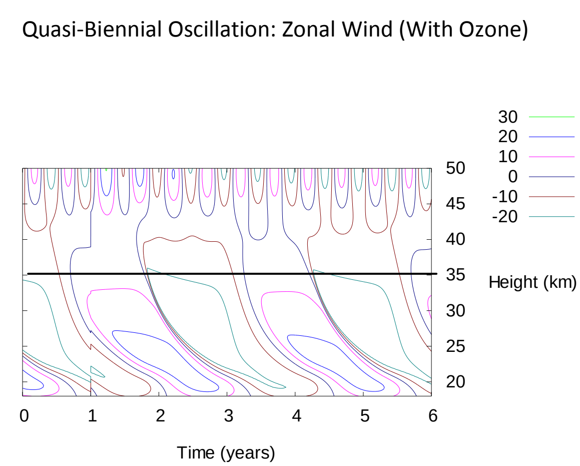
While the other terms besides $\frac{A\overline{\gamma_z}}{N^2}$ in equation 4.2 are important, they will not have a serious impact in the simulation because B and C are neglected in the simulation for simplicity. The one part worth acknowledging is the idea of the dynamically controlled region versus the photochemically controlled region and its implications on damping. In the dynamically controlled region below 35km altitude, $\omega \gg B$. This means that the primary source of the perturbation in the ozone mixing ratio will be wave fronts advecting ozone. Moreover, $\frac{\alpha B+AC}{\omega^2} \ll 1$, so equation 4.2 is simplified to show that there is less photochemical damping in this region. At around 35km altitude, we enter the photochemically controlled region. Here, wave fronts have less control on advection than the Chapman reactions and $\omega \ll B$. This causes the last term of equation 4.2 to have a noticeable impact on damping and changes the damping of the upward propagating Kelvin waves.

IV. RESULTS

To determine effects of various parameters on the QBO, a computer model was used running in C++. The first item tested was the effect of ozone on the QBO. In doing this, there were two trials performed: one without ozone and one with ozone. The first trial was performed without ozone and therefore had A(z)=B(z)=C(z)=0. The second trial was run with ozone, but was simplified such that only the ozone heating term appeared (A(z)=.25, B(z)=C(z)=0). The graphs of each trial were generated by WGNUPlot from the data and are shown below.



(a)



(b)

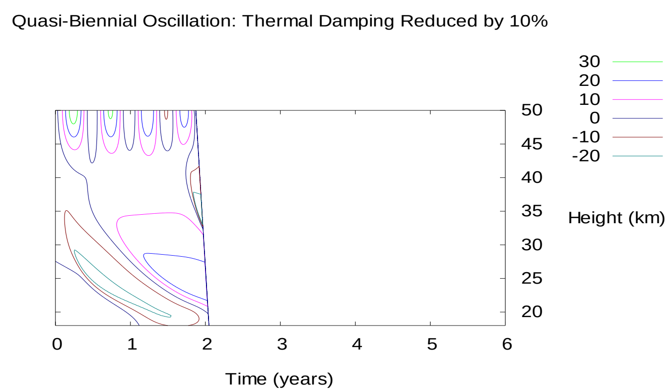
Figure 5: a. QBO without Ozone, b. QBO with Ozone

In these graphs, the contour colors represent speeds of zonal winds measured in meters per second. For this, westerly winds are shown as positive and easterly winds are shown as negative.

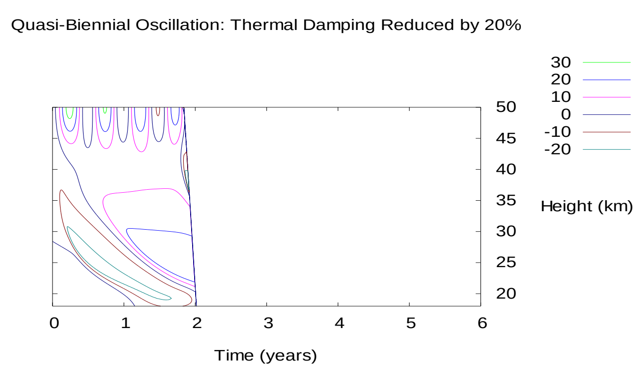
Before I discuss the differences between these graphs, I would like to point out the appearance of the fundamental parts of the QBO in them. First, look at the times when there is no zonal wind (u=0m/s) at the lower boundary of the plot (z=18km). One of these points occurs slightly after two years, another comes at about 3.3 years and then the next is at about 4.5 years. The full oscillation, which includes an easterly and a westerly regimes, therefore lasts slightly under 2.5 years in both the case with ozone and the one without ozone. This is slightly less than 30 months which is very close to the average of 27 months that has been observed. Also note how the higher zonal winds do not extend above approximately 35km nor below 20km. This fits with the observations stated in the introduction and shown in figure 2 that the QBO regimes start at about 35km and propagate down to 23km and are dissipated away quickly below that. Lastly, look at the dominance of easterlies at altitudes between 25km and 40km in terms of how long the regime lasts. This dominance fits the trend of figure 2 which shows that the model is in step with measurements there as well. Note, however, that the regimes at lower levels in figure 5 are approximately equal in length which does not fit with the observations of figure 2. All of this shows that this model gives a good approximation, but does not precisely replicate the patterns of the observed QBO.

These two plots, however, do have clear differences. First, look at the horizontal line drawn at 35km elevation on both plots. Along this line it is clear that the QBO model with ozone has higher winds at higher elevations. This is a consequence of the $\frac{A\overline{\gamma}_z}{N^2}$ term in equation 4.2. When A=0, as in the plot without ozone, this term doesn't have an effect. However, in the plot with ozone this term does have an effect. Looking at figure 4, we see that $\overline{\gamma_z} > 0$ for altitudes up to about 35km. Given the fact that this has opposite sign of α, the thermal damping term in equation 4.2, the effect of ozone in this model will be to lower damping below this altitude and allow Kelvin waves to propagate further upwards before they are dissipated and converted into zonal winds. This leads to higher zonal winds at higher altitudes as shown in figure 5. The second difference is in the speed of the winds at lower elevations around 20km to 25km. This difference also relates to the effect of the ozone heating term. At lower elevations $\overline{\gamma_z} > 0$. From equation 4.2, this should lead to less damping and explains why we see longer periods of strong zonal winds at these lower elevations. If a wave is weakly damped, its amplitude will be reduced at a lower rate and it will be able to contribute energy to its surroundings for a longer period of time. In addition, these plots show that ozone can contribute to the QBO in such a way that it may help explain why models using only Rossby-Gravity and Kelvin Waves and neglecting ozone fail to create a highly accurate representation of the QBO.

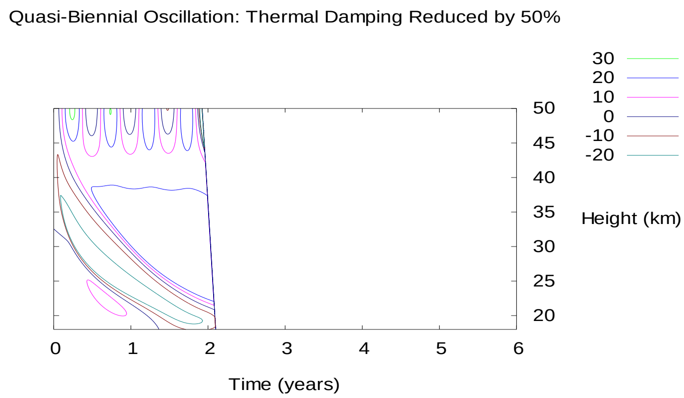
In the next part of the investigation we looked at the effect of the magnitude of thermal damping on the QBO. To investigate this, the computer model was set with the ozone coefficients as A=.25, B=C=0 to allow for comparison of the plots with altered thermal damping coefficients to the previous plot with ozone. Next, the thermal damping term, α, was reduced or increased by a factor that served as the experimental variable. For this analysis, I will first show the graphs for when the thermal damping term is lowered.



(a)



(b)



(c)

Figure 6: a. Thermal Damping Reduced by 10%, b. Thermal Damping Reduced by 20%, c. Thermal Damping Reduced by 50%

From these three graphs there are some key features that appear. First, we see larger zonal winds occurring at higher levels as the damping is decreased. Second, as the thermal damping decreases, westerly wind regimes are present for much larger time periods than easterly wind regimes at higher altitudes. Lastly, we see an increase in the length of the period of the oscillation. The presence of larger zonal winds at higher levels can be explained. These larger winds at higher levels occur because the lower levels of damping take away less of the upward propagating Kelvin and Rossby Gravity waves' energy as they propagate upward. Therefore, more of this energy can be transferred from the waves into the zonal winds at higher levels. As for the second observation, it results from a difference in the equations for the imaginary part of the wave numbers of Kelvin and Rossby-Gravity waves. This was given in equation 4.2 for the Kelvin wave, and can be simplified to $Im(m) = \frac{Nk}{2\omega^2}\left(\alpha - \frac{A\overline{\gamma_z}}{N^2}\right)$ given $B=0$ for our model. As for the Rossby-Gravity waves, it is shown by Cordero, Nathan and Echols in their 1998 paper that the relation is $Im(m) = \frac{N(\beta+\omega k)}{2\omega^3}\left(\alpha - \frac{A\overline{\gamma_z}}{N^2}\right)$ when $B=0$. These equations can be used along with the values given on page 435 of Holton's 2004 textbook to fully explain this effect. To start the analysis, the equations for the imaginary part of the waves are written in a general form used by Cordero, Nathan and Echols in their 1998 paper:

$$Im(m) = \frac{-Re(m)}{2\omega}\left(\alpha + \frac{1}{\omega^2}\left(\frac{A\overline{\gamma}_z\omega^2}{N^2}\right)\right) \quad (5.1)$$

Here, I have allowed $B=C=0$ and have $Re(m)$, and ω as being different for Kelvin and Rossby-Gravity Waves. These are the only differences in this equation for Kelvin and Rossby-Gravity waves given that the ozone coefficients B and C were neglected by this model. According to Cordero, Nathan and Echols 1998, the equations for the real part of the Kelvin and Rossby-Gravity Waves are, respectively:

$$Re(m)_K = \frac{-Nk}{\omega} \quad (5.2a)$$

$$Re(m)_{RG} = \frac{N}{\omega^2}(\beta - \omega k) \quad (5.2b)$$

Now, note from equation 5.1 that these real parts of the wave number divided by two times their angular frequencies serve as the coefficient to α, the thermal damping coefficient, and therefore computing them for approximate observed values will give insight into how much of an effect increased or decreased thermal damping will affect their amplitudes. So, using the following values, I will compute $\frac{Re(m)}{2\omega}$ for both the Rossby-Gravity waves and the Kelvin waves.

Table I: Values for Observed Kelvin and Rossby-Gravity Waves [27]

Wave Type	**Period (days)**	**Angular Frequency (ω) (rad/day)**	**Wave Number (k)**[28]
Kelvin	15	.418	1.5/a
Rossby-Gravity	4	1.57	4/a

From using these values along with $\beta = \frac{2\Omega}{a} \approx \frac{12.6\ rad/day}{a}$ for equatorial motion equations 5.2a and 5.2b, we get[29] $Re(m)_K \approx 3.6\left(\frac{N}{a}\right)$, and $Re(m)_{RG} \approx 2.6\left(\frac{N}{a}\right)$. Moreover, using these values again, we find $\frac{Re(m)_K}{2\omega} \approx 4.3\left(\frac{N}{a}\right)$, and $\frac{Re(m)_{RG}}{2\omega} \approx .82\left(\frac{N}{a}\right)$. From equation 5.1, we see that this means the Kelvin waves have a larger coefficient to the thermal damping term, and therefore decreasing the thermal damping should cause them to dominate at higher altitudes. This is consistent with figure 6 because as thermal damping is decreased westerly regimes which are caused in part by the damping of Kelvin waves dominate at higher altitudes.

The longer period of this oscillation can also be explained. With less damping, the higher winds start at higher altitudes. As was stated in the introduction, the QBO propagates downward due to a Doppler shift that causes waves of the existing

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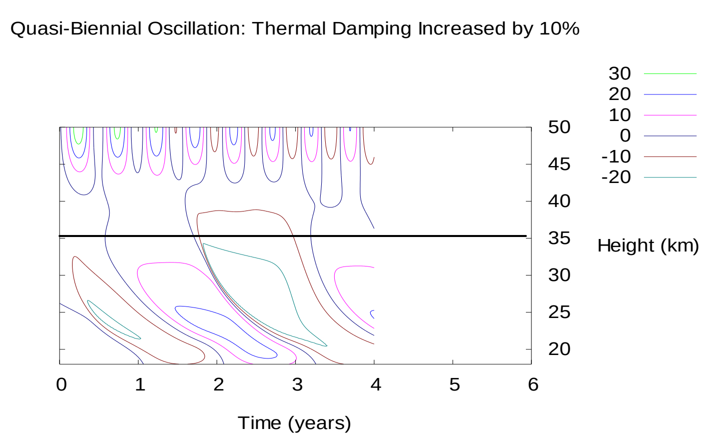
[27] Holton, 2004 p. 435, modified for ϕ=0.

[28] Here a=mean radius of the earth, and is left in both for convenience and because the computation performed is done for a comparison, not an exact value.

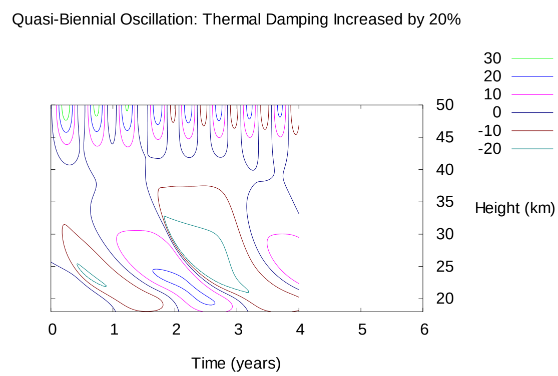
[29] Here units are identical for both and, given that we are only performing a comparison, they are neglected for neatness.

regime to be damped preferentially and not be able to propagate in significant amplitude beyond where there are high winds. So, if the higher winds start at higher altitude, the damping that prevents waves of the type of the existing regime from reaching a higher altitude will have to cover more vertical length to get to 23km where the winds are damped away. This extra length for which the damping must occur over will take more time and thus accounts for the longer period. In addition to the properties of the QBO, the model also revealed an interesting fact about the necessity of thermal damping in the QBO: at a 70% reduction in the coefficient for thermal damping, the QBO ceased to exist.

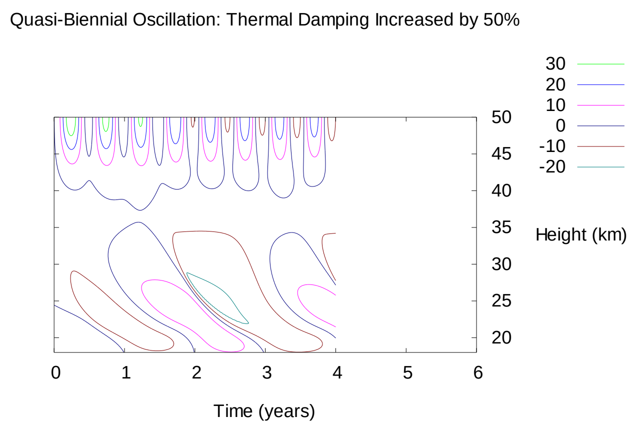
Lastly, I explored the effects of increased thermal damping. The experimental set-up was almost identical to the one done for reduced thermal damping, except that the coefficient for the thermal damping, α, was increased instead of being decreased. In doing so, the following plots were produced:



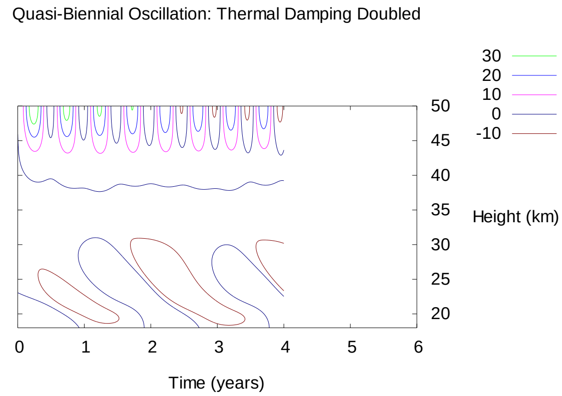
(a)


Quasi-Biennial Oscillation: Thermal Damping Increased by 20%
30
20
10
0
-10
-20
50
45
40
35
30
25
20
Height (km)
0
1
2
3
4
5
6
Time (years)

(b)


Quasi-Biennial Oscillation: Thermal Damping Increased by 50%
30
20
10
0
-10
-20
50
45
40
35
30
25
20
Height (km)
0
1
2
3
4
5
6
Time (years)

(c)



(d)

Figure 7: a. Damping Increased by 10%, b. Damping Increased by 20%, c. Damping Increased by 50%, d. Damping Doubled

In analyzing these graphs, many patterns fit the trends established in the earlier part involving lowered thermal damping. The first pattern is that as the damping is increased, the larger zonal winds occur at a much lower and more limited range of altitudes. Second, as damping is increased, the easterly regimes begin to show higher winds speeds than westerly regimes. Lastly, the period of the oscillation decreases as the damping is increased. For the first of these, it can be seen that the higher damping will cause an upward propagating wave such as a Kelvin or Rossby Gravity wave to lose more of its energy as it propagates, so it will have less energy that can be converted at higher altitudes to zonal winds. A similar explanation follows for this second observation compared to what was shown for decreasing the thermal damping in the previous part of this report. Once again, it depends on the imaginary part of the vertical wave number, only this time α is increasing instead of decreasing. This leads to the higher coefficient for the imaginary part of the wave number of the Kelvin waves causing the damping to increase more as α increases. Therefore, the westerly regimes (to which the Kelvin waves contribute) should occur at lower altitudes, decrease in wind speed and last for less time. All three of these appear in the figures shown. The last observation about the period of the oscillation merits a similar explanation for the period differences in the previous part involving lowered thermal damping. In the case of high thermal damping, waves are damped more quickly which causes higher winds to occur at lower levels. This causes regimes to reach 23km of altitude more quickly where they are damped away. Also, the increased damping reduces the amplitude of waves more quickly which also lowers the period because it takes less time to dissipate the upward propagating Kelvin and Rossby-Gravity waves that cause the QBO. In addition to these properties, I also found that around triple the thermal damping there ceases to be a QBO.

V. SUMMARY AND CONCLUSION

In this investigation, we used a model based on the standard atmospheric equations and perturbation theory to explain the QBO and then used a program to show the effects of increased or decreased damping as well as ozone on the QBO. In the process, this report showed the basic dynamics of the QBO. In analyzing these dynamics, mathematical methods were used to make sense of some of the results as well as to give some background to the model used. Moreover, in the Results section, there was shown to be a difference between the wind pattern with ozone and the wind pattern without ozone which was in accordance with the conclusions reached by Echols and Nathan in their 1996 paper and later by Cordero, Nathan and Echols in their 1998 paper. The difference between these models was that the one with ozone had higher wind speeds at higher altitudes and at lower altitudes of the region between 18km and 40km, while the wind speeds were similar for both models in the middle-altitude region. It was shown that these different wind speeds are caused by a sign change in $\overline{\gamma_z}$ which led to a change in the imaginary part of the wave numbers and the damping of the vertically propagating waves.

While there was a difference in the graphs of the QBO which included ozone and those which did not, it would have likely been greater if the model had been modified to include a non-constant value for the ozone heating coefficient $A(z)$, or non-zero values for $B(z)$ and $C(z)$. In this sense, this model was somewhat simplistic and fails to give the complete extent of the impact of ozone on the QBO. However, qualitatively this model should give the reader an idea that, even when taken into marginal account, ozone has a visible impact on the QBO.

Besides the effect of ozone on the QBO, we further investigated the effect of thermal damping. This was noted in the Introduction to have a significant role on the QBO. In fact, the QBO is primarily caused by damping, of which, thermal damping plays the largest role. It came as an interesting result that relatively small differences in thermal damping produce relatively large effects on the oscillation. All of this, however, must be taken into context. In this model, thermal damping was not adjusted by putting in different values of atmospheric parameters that are within a range that could reasonably occur; instead, it was adjusted by introducing a scaling factor that directly changed the value of the coefficient representing thermal damping in the governing equations, α. This change makes our results only valid in a hypothetical atmosphere. In doing this, it was found that the QBO may only occur for a limited range of values for thermal damping. Namely, in order for the QBO to occur, $.3\alpha_0 \leq \alpha \leq 3\alpha_0$, where α_0 is the standard value of the radiational cooling coefficient for the atmosphere. This can also be seen as attempting to determine a pattern for the oscillation that may give clues as to what might happen if there were drastic change in the atmosphere as a result of current patterns such as global warming, or other future climate change. While such changes are outside of the scope of this report, the potential results for these could be significant given the differences between the graphs showing variations in damping of ten percent (figure 6a and 8a) versus the QBO graph with ozone (figure 5b) which they are derived from.

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